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International Journal of Heat and Mass Transfer 43 (2000) 4205–4212

International Journal of
**HEAT and MASS
TRANSFER**

www.elsevier.com/locate/ijhmt

Technical Note

Thermodynamic analysis of thermomechanical coupling in Couette flow

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Received 22 February 1999; received in revised form 26 January 2000

1. Introduction

Couette flow provides the simplest model for the analysis of heat transfer for flow between parallel plates and between two coaxial cylinders. Design of such devices with the consideration of the thermomechanical coupling of viscous heat generation and Couette flow is important since they are used in lubrication, polymer and food processing, and viscometry [1–4]. The tangential annular flow is a good model for a journal and its bearing, in which, one surface is stationary while the other is rotating and the clearance between the surfaces is filled with an incompressible fluid that is mostly a lubricant oil of high viscosity. For such a system, the viscous-energy-dissipation as a heat source is used in the energy equation to predict the temperature distribution in the narrow gap of a Couette device. Temperature profiles in plane and circular Couette flows with and without pressure gradient, and with various thermal boundary conditions have been presented previously [1–6]. The thermodynamic analysis of a Couette flow is also important since the heat transfer and friction are accompanied by the entropy generation due to irreversibilities that are related to the amount of energy dissipation in the system [7–9]. In a previous work [10], the entropy in a circular Couette flow of temperature-dependent materials have been examined and found that the narrowing the gap of Couette increases the entropy generation shar-

ply. The objective of this study is to analyze the thermodynamics aspect of the plane and circular Couette flows with asymmetric wall temperatures for an incompressible fluid with temperature independent viscosity and thermal conductivity. In the analysis, the entropy generation has been calculated using the velocity and temperature profiles for a steady state, developed laminar flow. The effects of the pressure gradient and the Brinkman number on the volumetric entropy generation and the irreversibility distribution ratio have been determined and displayed graphically for both geometries.

2. Thermodynamic analysis

A Couette flow may describe the heat transfer in a journal and its bearing, in which, one surface is stationary while the other is rotating. If the gap between the surfaces is narrow in comparison with the radius of the bearing, the geometry can be treated as two parallel flat plates. In the gap filled with a viscous fluid, the temperature rises and the temperature profile may be developed due to friction. Since the temperature rises, the rate of heat transfer through the surfaces may be considerable [2,5] and an excessive entropy may be generated even at the moderate flow velocities [8,10]. The Gouy–Stodola theorem links the lost available energy to the entropy generation, thus relating economic implications of the different irreversibilities due to the operational and design conditions for a desired task of the system [7–9]. The rate of entropy

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Nomenclature

a, b, c	parameters (Eq. (6))
A, B	parameters (Eqs. (3) and (11), respectively)
Be	Bejan number
Br	Brinkman number (Eq. (6))
k	thermal conductivity of fluid ($\text{W m}^{-1} \text{K}^{-1}$)
r	radial direction
n	r_i/r_0
R	r/r_0
Re	Reynolds number ($Re = wr_0^0/\nu$)
S'''	volumetric rate of entropy generation ($\text{W m}^{-3} \text{K}^{-1}$)

T	temperature (K)
u	velocity (m s^{-1})
U	dimensionless velocity (u/u_1)
Y	y/H
w	angular velocity (s^{-1})

Greek symbols

ν	kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
ρ	density (kg m^{-3})
θ	dimensionless temperature

generation for a steady, Newtonian Couette flow is calculated in the following sections.

2.1. Plane Couette flow

The rate of entropy generation per unit volume S''' of an incompressible, Newtonian fluid for Cartesian coordinates is given by [8]

$$S''' = \frac{k}{T^2} \left(\frac{dT}{dy} \right)^2 + \frac{\mu}{T} \left(\frac{du}{dy} \right)^2 \quad (1)$$

Eq. (1) indicates that the velocity and temperature profiles are necessary to determine the volumetric entropy generation rate.

Fig. 1 shows the plane Couette flow with the following boundary conditions:

$$\begin{aligned} y = H & \quad u = u_1 & \quad T = T_1 \\ y = 0 & \quad u = 0 & \quad T = T_2 \end{aligned}$$

Equation of motion for fully developed flow in the x direction with pressure gradient is given by [2,6]

$$-\frac{d}{dy} \left[\mu \left(\frac{du}{dy} \right) \right] = \left(-\frac{dP}{dx} \right) \quad (2)$$

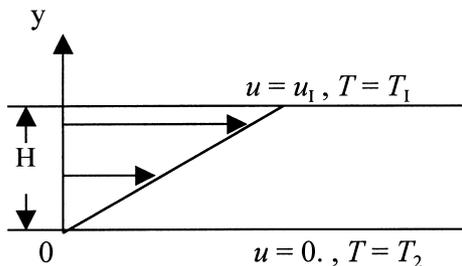


Fig. 1. The plane Couette flow.

Using the boundary conditions, Eq. (2) can be solved by integrating twice, and the velocity profile is obtained as

$$U = \frac{u}{u_1} Y [1 + A(1 - Y)] \quad (3)$$

where

$$A = \frac{(-dp/dx)H^2}{2\mu u_1}; \quad Y = \frac{y}{H}$$

The velocity gradient du/dy is obtained from Eq. (3), and is given by

$$\frac{du}{dy} = \frac{(-dP/dx)}{2\mu} (H - 2y) + \frac{u_1}{H} \quad (4)$$

The energy equation for laminar and hydrodynamically developed flow is [2]

$$\frac{d^2T}{dy^2} = \frac{\mu}{k} \left(\frac{du}{dy} \right)^2 \quad (5)$$

Substitution of Eq. (4) into Eq. (5) yields the following temperature distribution

$$\begin{aligned} \theta = \frac{T - T_2}{T_1 - T_2} = & \frac{(-dP/dx)Y}{12k(T_1 - T_2)} [aH^4(1 - Y^3) \\ & - 2bH^3(1 - Y^2) + 6cH^2(1 - Y)] \\ & + Y \left[1 + \frac{1}{2}Br(1 - Y) \right] \end{aligned} \quad (6)$$

where

$$a = \frac{(-dP/dx)}{\mu}$$

$$b = \frac{(-dP/dx)H}{\mu} + \frac{2u_1}{H}$$

$$c = \frac{(-dP/dx)H^2}{4\mu} + u_1$$

$$Br = \frac{\mu u_1^2}{k(T_1 - T_2)}$$

Br is the Brinkman number, which is a measure of the viscous heating as compared to the heat conducted through the gap of Couette device. The temperature gradient can be obtained from Eq. (6), and expressed by

$$\frac{dT}{dy} = \frac{(-dP/dx)}{12k} [aH^3(1 - 4Y^3) - 2bH^2(1 - 3Y^2) + 6cH(1 - 2Y)] + \frac{T_1 - T_2}{H} \left[1 + \frac{1}{2}Br(1 - 2Y) \right] \quad (7)$$

Inserting Eqs. (4) and (7) into Eq. (1) yields an expression for the volumetric entropy generation rate for a Couette flow between parallel flat plates.

Using the data presented in Table 1, the velocity and temperature profiles have been calculated for $H = 0.005$ m. The Brinkman number is changed in the range of -2 to 8 , while A has the three values that are -3.0 , 0 , and 3.0 . The dimensionless velocity profile U is shown in Fig. 2. For the case of $(-dp/dx) = 0$, known as simple Couette flow, the velocity is linear across the fluid. For a negative pressure drop the velocity is positive, and for a pressure increase, the velocity can become negative, leading to back flow. The

Table 1

The data used in calculations of the velocity and temperature profiles, and the rate of entropy generations^a

Plane Couette	Circular Couette
$H = 0.005$ (m)	$r_0 = 0.02$ (m)
$u_1 = 5$ (ms ⁻¹)	$r_1 = 0.019$ (m)
$T_2 = 300$ (K)	$w = 1048$ (s ⁻¹)
	$T_i = 300$ (K)
	$Re = 5000$

^a Fluid: unused engine oil, constant physical properties [12]: $k = 0.14$ (W m⁻¹ K⁻¹), $\nu = 0.839 \times 10^{-4}$ (m² s⁻¹), $\rho = 864.04$ (kgm⁻³).

point of reversal is that point at which $du/dy = 0$ at $y = 0$. This occurs when $-dp/dx = -2\mu u_1/(H^2)$ [6].

Fig. 3 shows the dimensionless temperature profiles θ for $T_2 = 300$ K and $-2.0 < Br < 8.0$ with and without the pressure gradient. The rise of temperature, in the middle part of Couette device, is considerably large for high values of Br [5,6]. Spatial distribution and the location of maximum temperature along the gap are highly affected by the pressure gradient.

2.2. Circular Couette flow

The entropy generation rate for an incompressible Newtonian fluid held between two coaxial cylinders is given by [8]

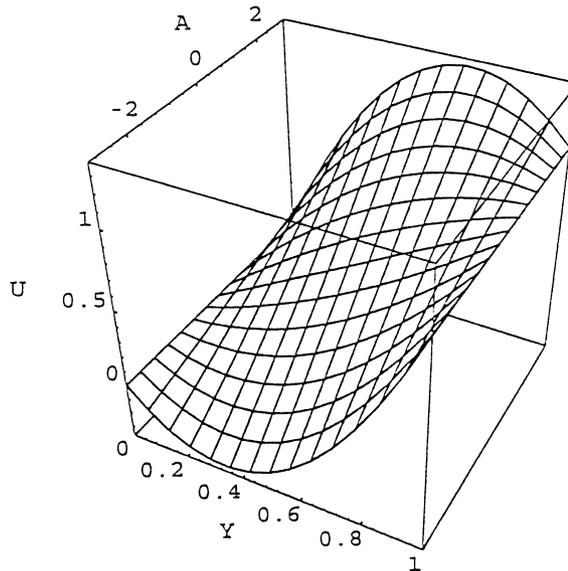


Fig. 2. Dimensionless velocity profile U for the plane Couette flow for $H = 0.005$ m.

$$S_{\text{gen}}''' = \frac{k}{T^2} \left(\frac{dT}{dr} \right)^2 + \frac{\mu}{T} \left[r \frac{d}{dr} \left(\frac{u_\theta}{r} \right) \right]^2 \quad (8)$$

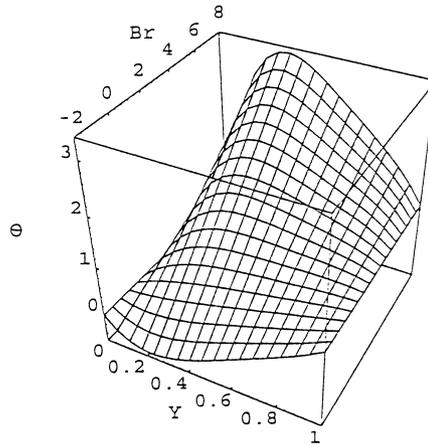
The circular Couette flow is shown in Fig. 4. The flow between concentric cylinders is in the direction θ only, and it satisfies that $u_\theta = u_\theta(r)$, $T = T(r)$. The inner cylinder is stationary while the outer one is rotating with an angular velocity w . It is assumed that the annular flow is laminar and there are no end effects, and the steady state is reached with the controlled

angular velocity. The solutions of the velocity and temperature distributions of θ direction are available [5,6]

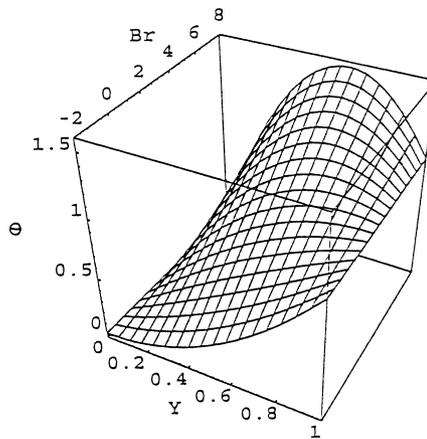
$$\frac{u_\theta}{wr_0} = \frac{1}{R} \frac{(r^2 - r_i^2)}{(r_0^2 - r_i^2)} \quad (9)$$

where

$$R = \frac{r}{r_0}$$

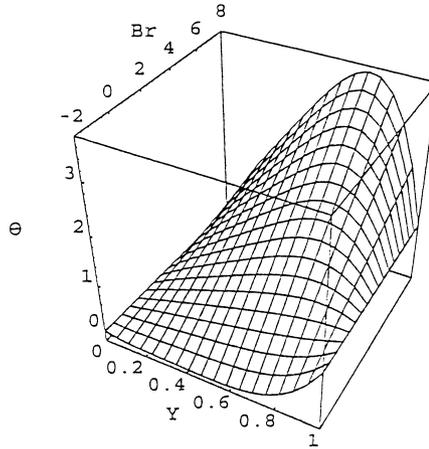


(a) A = -3.0



(b) A = 0.0

Fig. 3. Dimensionless temperature profile θ , for the plane Couette flow for $u_1 = 5.0 \text{ ms}^{-1}$, $H = 0.005 \text{ m}$.



(c) A = 3.0

Fig. 3 (continued)

The second term on the right-hand side of Eq. (8), related to the velocity gradient, can be obtained from Eq. (9), and expressed by

$$\frac{\mu}{T} \left[r \frac{d}{dr} \left(\frac{u_0}{r} \right) \right]^2 = \frac{\mu}{T} \frac{4u_1^2 r_0^2 r_i^4}{r^4 (r_0^2 - r_i^2)^2} \tag{10}$$

The temperature profile is given by [5].

$$\theta = \frac{T - T_i}{T_0 - T_i} = (B + 1) \left(1 - \frac{\ln R}{\ln n} \right) + B \left(\frac{\ln R}{n^2 \ln n} - \frac{1}{R^2} \right) \tag{11}$$

with the dimensionless quantities

$$n = \frac{r_i}{r_0}; \quad B = Br \frac{n^4}{(1 - n^2)^2} \quad \text{and} \quad Br = \frac{\mu w^2 r_0^2}{k(T_0 - T_i)}$$

Br is the Brinkman number for the annulus. Eqs. (9) and (11) satisfy the following boundary conditions:

$$\begin{aligned} R = n & \quad u_\theta = 0 & \quad \theta = 0 \\ R = 1 & \quad u_\theta = wr_0 & \quad \theta = 1 \end{aligned}$$

The temperature gradient can be obtained from Eq. (11) and given by

$$\frac{dT}{dr} = (T_0 - T_i) \left(\frac{2Br_0^2}{r^3} + \frac{B}{rn^2 \ln n} - \frac{B+1}{r \ln n} \right) \tag{12}$$

Using the data presented in Table 1, the temperature profile has been calculated in the region of $1 < Br < 8$ for $Re = 5000$, $T_i = 300$ K, and $w = 1048$ s⁻¹, and shown in Fig. 5.

By inserting Eqs. (10) and (12) into Eq. (8), an ex-

pression for the volumetric entropy generation rate of the circular Couette flow can be obtained. The first terms on the right-hand sides of Eqs. (1) and (8) show the entropy generation due to the heat transfer $S''_{\Delta T}$, while the entropy generation due to the fluid friction $S'''_{\Delta P}$ is shown by the second terms, hence the rate of entropy generation expression has the following basic form

$$S''' = S''_{\Delta T} + S'''_{\Delta P} \tag{13}$$

The volumetric entropy generation rate is positive and finite as long as temperature or velocity gradients are present in the medium. The irreversibility distribution ratio is defined as

$$Be = S''_{\Delta T} / S''' \tag{14}$$

and was named as the Bejan number, Be [11]. Be = 1 is the limit at which all the irreversibility is due to the

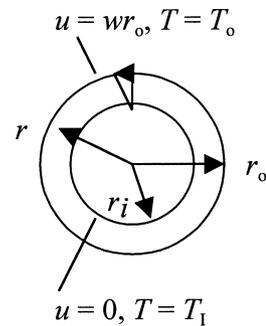
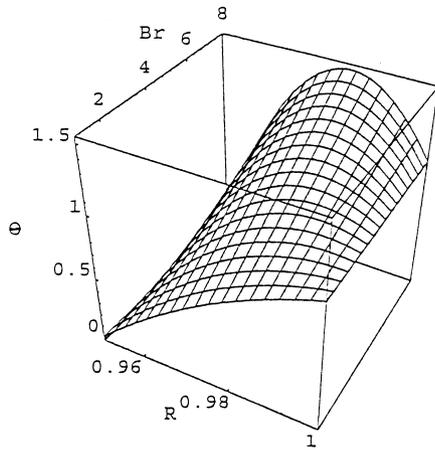


Fig. 4. The circular Couette flow.



$$w = 1048 \text{ s}^{-1} ; Re = 5000$$

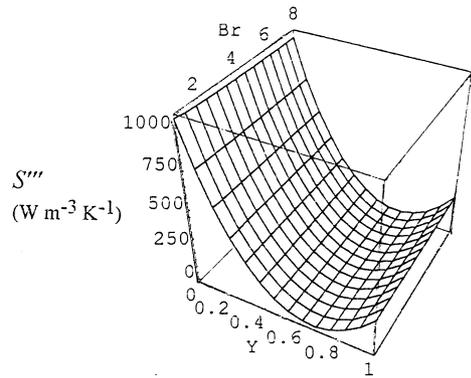
Fig. 5. Dimensionless temperature profile θ for the circular Couette flow for $T_i = 300 \text{ K}$, $r_0 = 0.02 \text{ m}$, $r_i = 0.019 \text{ m}$, $w = 1048 \text{ s}^{-1}$, $Re = 5000$.

heat transfer. The irreversibility due to the heat transfer dominates when $Be \gg \frac{1}{2}$, while $Be \ll \frac{1}{2}$ is the case where the irreversibility due to the friction dominates [8].

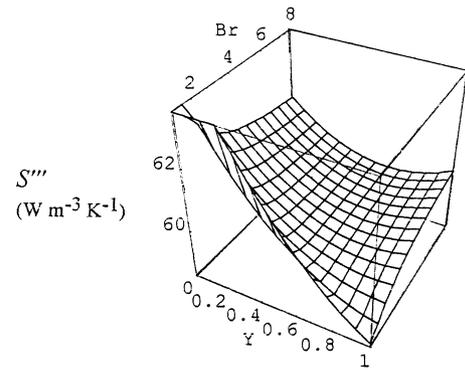
3. Results and discussion

Unused engine oil is used as the fluid in the gap of Couette devices. The data for the Couette devices and the physical properties of the fluid that are assumed to be temperature independent are given in Table 1. For the plane Couette, the effects of A and Br on the volumetric entropy generation are shown in Fig. 6. The values of S''' are influenced mainly by the pressure gradient, and produce minimum values around $Y = 0.7$ and 0.4 with $A = -3.0$ and 3.0 , respectively. When $A = 0.0$, the values of S''' are rather small, and varies with both A and Br . The irreversibility distribution ratio Be is shown in Fig. 7 for $T_2 = 300 \text{ K}$. The domination of the entropy generation due to friction is apparent. The peak values of Be appear in the low entropy generation regions when $A = -3.0$ and 3.0 , as seen in Fig. 6. Without the pressure gradient ($A = 0$) entropy generation is relatively small.

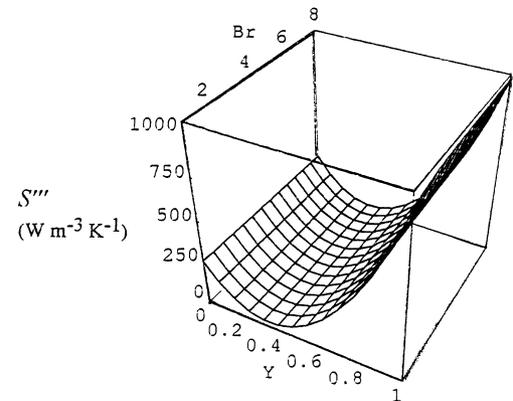
For circular Couette, the Reynolds number ($Re = wr_0^2/\nu$) at the transition from laminar to turbulent flow



(a) $A = -3.0$



(b) $A = 0.0$



(c) $A = 3.0$

Fig. 6. The volumetric entropy generation S''' for the plane Couette flow for $u_1 = 5.0 \text{ ms}^{-1}$, $H = 0.005 \text{ m}$.

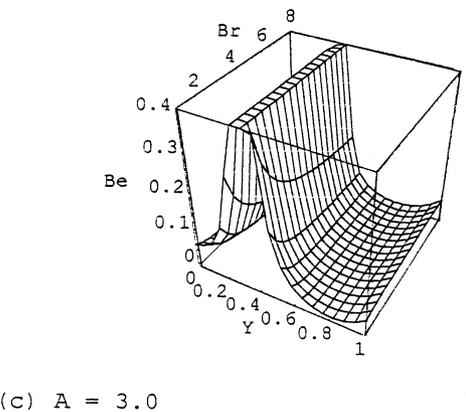
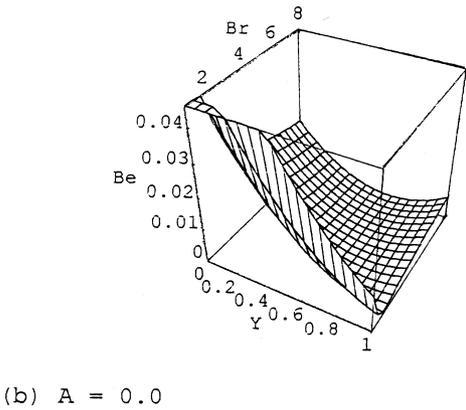
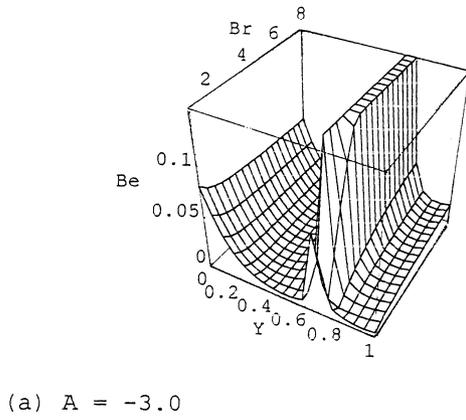


Fig. 7. The Bejan number Be for the plane Couette flow for $u_1 = 5.0 \text{ ms}^{-1}$, $H = 0.005 \text{ m}$.

is strongly dependent on the ratio of the gap to the radius of the outer cylinder, $1 - n$. The critical Reynolds number reaches a value of about 50,000 at $1 - n = 0.05$ [5]. Therefore, the Reynolds number less than 50,000 and $1 - n = 0.05$ are adapted in the calculations. With the data adapted in Table 1, n becomes

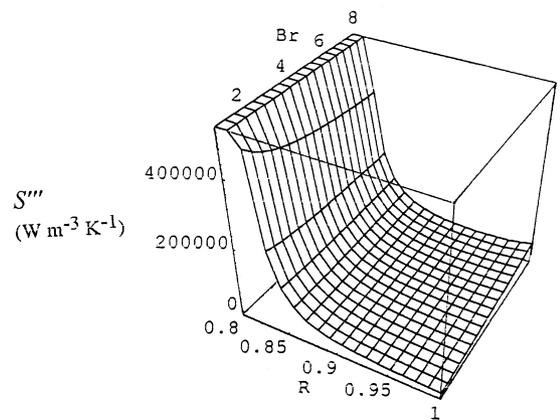
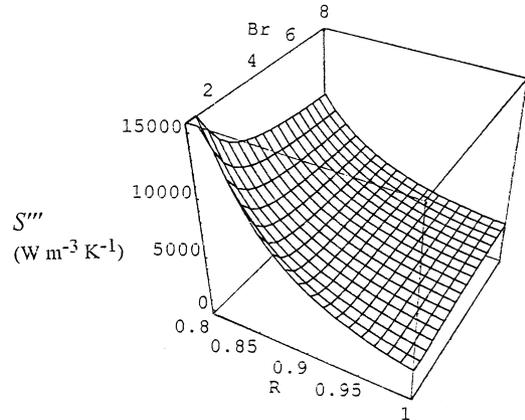
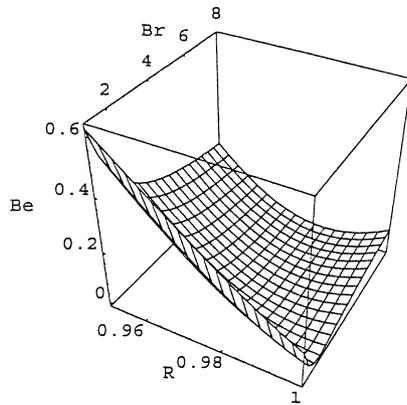


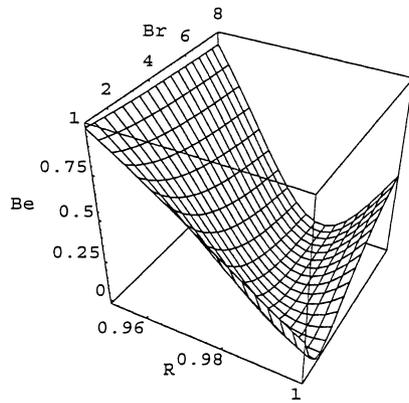
Fig. 8. The volumetric entropy generation S''' for the circular Couette flow for $T_i = 300 \text{ K}$, $r_0 = 0.02 \text{ m}$, $r_i = 0.019 \text{ m}$.

0.95. The angular velocity w and the temperature difference $T_0 - T_i$ are determined from the specified values of Re and Br , and used in the entropy generation calculations, which are shown in Fig. 8. At high Re , the effect of Br on the entropy generation is minimal, while Br has some effect on S''' at low Re . The irreversibility distribution ratio Be for $Re = 5000$ and $40,000$, and for $T_i = 300 \text{ K}$ is shown in Fig. 9. The increase of Be indicates a competition of the irreversibilities caused by the heat transfer and friction. At high Re , the distribution of Be is relatively more uniform than that at low Re .

With various operational conditions such as, the gap of Couette device, the Brinkman number, Re , and the boundary conditions, the volumetric entropy generation can be calculated easily by Eqs. (1) and (8). Such calculations can display the level of entropy generation



(a) $w = 1048 \text{ s}^{-1}$; $Re = 5000$



(b) $w = 8390 \text{ s}^{-1}$; $Re = 40000$

Fig. 9. The Bejan number Be for the circular Couette flow for $T_i = 300 \text{ K}$, $r_0 = 0.02 \text{ m}$, $r_i = 0.019 \text{ m}$.

for a required task from an existing Couette with the specified operating conditions, and therefore help to analyze the system thermodynamically. Since the irreversibility is related to loss of energy, the thermodynamic analysis can facilitate the optimal operating con-

ditions that produce less entropy for an existing design.

Acknowledgements

The author is grateful to King Fahd University of Petroleum and Minerals for the support provided.

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